Quantum Criticality and Novel Phases: A panel discussion.

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Physicists gathered in august at Dresden for a conference about "Quantum Criticality and Novel Phases". As one part of the meeting, nine panelists hosted an open and free-wheeling discussion on the topic of the meeting. This article outlines the discussions that took place during at this panel-meeting on the afternoon of August 3rd, 2009.

1 Introduction Eighty years ago, physicist Paul Dirac, reflecting on the new quantum theory he had played such an important part in developing, wistfully remarked that

"the underlying physical laws necessary for the mathematical theory of . . . physics and the whole of chemistry are thus completely known, and the difficulty is only that . . . these laws lead to equations much too complicated to be soluble" P. A. M. Dirac ∼1929.[1]

Discoveries spanning eight decades have revealed Dirac's remark to be one of the great physics understatements of all time, for understanding the link between the quantum micro-world and our emergent macroscopic world proves a singular challenge. Today, we know that the rules of quantum mechanics endow matter with a propensity to develop unexpectedly simple, yet completely new kinds of collective behavior. From the practical perspective of the material physicist, emergence means that the periodic table is a forge of fabulous potential, from which wholly new kinds of material can be crafted, high temperature superconductors, materials with new kinds of multi-functional behavior such as multiferrocity and materials with possibilities that we have yet to discover or even imagine. But its easy to get lost, and guiding principles are invaluable.

One such principle that has appears increasingly fruitful, is to seek materials that lie at the point of instability between one phase and another: this point is called a "quantum phase transition" (QPT)[2] . Conventional phase transitions are driven by thermal motions at finite temperature. Quantum phase transitions occur at the point where the transition temperature is tuned to absolute zero. At absolute zero, thermal motion vanishes, yet quantum phase transitions are far from static, and can be likened to a melting phenomenon driven by the zero point quantum motion that arises from Heisenberg's uncertainty principle. Such zero point motion is particularly important at a second order quantum phase transition, where fluctuations in the order parameter develop an infinite correlation length and an infinite correlation time that engulfs the entire material: such a singular point of transition is a "Quantum Critical Point" (QCP)[3,2,4].

Experiments show that radically new kinds of metallic behavior develop when a quantum critical metal is warmed to a finite temperature[5,6] Such materials also have a marked propensity to nucleate new kinds of order. The "dome" of superconductivity in the phase diagram of high temperature cuprate superconductors is thought by many, to hide a quantum critical point, but the superconductivity is so robust that huge magnetic fields are required to strip away the superconductivity to reveal the underlying quantum critical point, and the idea is still controversial. Fortunately, similar situations occur in heavy fermion materials, such as $CeRhIn_5$, where superconductivity nucleates

around a pressure-tuned antiferromagnetic quantum critical point and can be removed by more modest magnetic field[7]. The important point is that quantum criticality appears to provide a vital way of inducing high temperature superconductivity and other novel material behavior; it is this potential, together with the dramatic transformations in metallic behavior that appear to accompany quantum criticality that motivate the research behind the the Dresden "Quantum Criticality and Novel Phases" conferences, discussed in the panel discussion reported here.

Nine panelists Meigan Aronson (Brookhaven National Laboratory and Stony Brook University, New York, USA), Piers Coleman (Rutgers University, New Jersey, USA), Philipp Gegenwart (University of Göttingen, Germany), Hilbert von Löhneysen (Karlsruhe University, Germany), Brian Maple (University of California, San Diego, USA), Suchitra Sebastian (Cambridge University, UK), T. Senthil (MIT, USA), Kazuo Ueda (Institute for Solid State Physics, Tokyo, Japan) and Tomo Uemura (Columbia University, New York, USA). came together at the conference for a wide-ranging discussion on this topic. As a prequel to the discussions, each participant posted their questions and thoughts on a wiki discussion site[8].

In reporting the discussions, I have not followed the original ordering of speakers, but instead, to group the discussions by topic. Any mistakes in the rendition of the ideas that were presented at the discussion are most likely my own, for which I apologize in advance. I am particularly indebted to those speakers who sent me notes on their discussion.

2 f-electrons as a route to wider understanding One of the persistent threads throughout the discussion, was the usefulness of f-electron materials as a research basis for studying quantum criticality and novel phases of strongly correlated materials. **Meigan Aronson** emphasized how so much of our understanding about quantum criticality and zero temperature phase transitions comes from systematic studies of f-electron based compounds, where the low energy/temperature range of the f-bands means that the stability of magnetic order can be tuned and fully suppressed by modest variations in pressure, composition and magnetic fields.

Kazuo Ueda agreed, and described how in Japan, the utility of f-electron research has recently been recognized by the Japanese Ministry of Education through the establishment of a distributed research network, consisting of sevean closely linked research consortia across Japan working on a wide-band of projects in f-electron physics. The network also has funds for several independent research projects for smaller research groups some of an applied nature, to link up with the consortium. One of the vital reasons, he said, for a distributed consortium, is that it makes it possible to share materials , skills, resources and high quality spectroscopy without them being concentrated at a single institution.

Meigan Aronson referred to the mounting evidence from the phase diagrams of systems with very different microscopic physics - such as organic conductors, and the new Fe-based superconductors - that unconventional superconductivity may generically occur near magnetic quantum critical points, suggesting a universality to the overall behavior which was originally found in f-electron based systems.

3 Strange Metals and Quantum Criticality One of the key topics, was strange metal behavior and its possible connection with quantum criticality. There was a wide ranging discussion about the meaning of quantum criticality. Questions were also raised about whether the linkage between bulk quantum criticality, and the development of strange metal behavior is indeed always apparent?

Hilbert von Lohneysen emphasized that even though a Quantum Phase Transition (QPT) is strictly speaking a zero-temperature instability, its experimental manifestations are clearly seen at finite temperatures. In metals these anomalies give rise to departures from canonical Landau Fermi liquid behavior that are often refered to as "non-Fermi-liquid" (NFL) behavior. In general terms, he argued, one *can* understand this finite temperature effect of a quantum critical point in terms of a quantum-classical mapping in which temperature sets the maximum time scale for coherent quantum processes[9,3,4,10] , introducing a finite system size L_τ in the time direction.

$$L_\tau = \frac{\hbar}{k_B T}.$$

According to this picture, he emphasized, one expects that the fan-shaped catchment area of a QCP will extend at most to temperatures where $\hbar/L_\tau$ remains small compared with other relevant energy scales, such as the Kondo scale $k_B T_K$.

Puzzlingly, though, in several systems, the NFL behavior associated with the QCP extends to temperatures up to or even in excess of T_K. For instance, in $\mathrm{CeCu_{5.9}Au_{0.1}}$ with $T_K \approx 5$K, the anomalous scaling exponent $\alpha = 0.75$ in the dc susceptibility $\chi(T)^{-1} = (A + BT^\alpha)$extends up n to 6 K [1]. This resembles, he remarked, the anomalous T-linear resistivity in the cuprate superconductors, extending up to $\approx$ 1000K in optimally doped $\mathrm{YBa_2Cu_3O_7}$. Both **Suchitra Sebastian** and **Hilbert von Lohneysen** questioned

> *whether the remarkably large temperature ranges over which strange metal behavior is observed is a signature of the asymptotic low temperature quantum criticality, or whether it is some other kind of separate precursor?*

For example, in quantum critical $\mathrm{YbRh_2Si_{2-x}Ge_x}$ a logarithmic specific heat $\frac{C}{T} \sim \frac{1}{T_0}\ln(\frac{T_0}{T})$ is seen over two decades of temperature, where $T_0 = 24K$, but at temperatures below $T^* \sim 0.3K$, the specific heat is seen to cross over to a power-law behavior $C/T \sim T^{-1/3}$[11] (See Fig. 1).

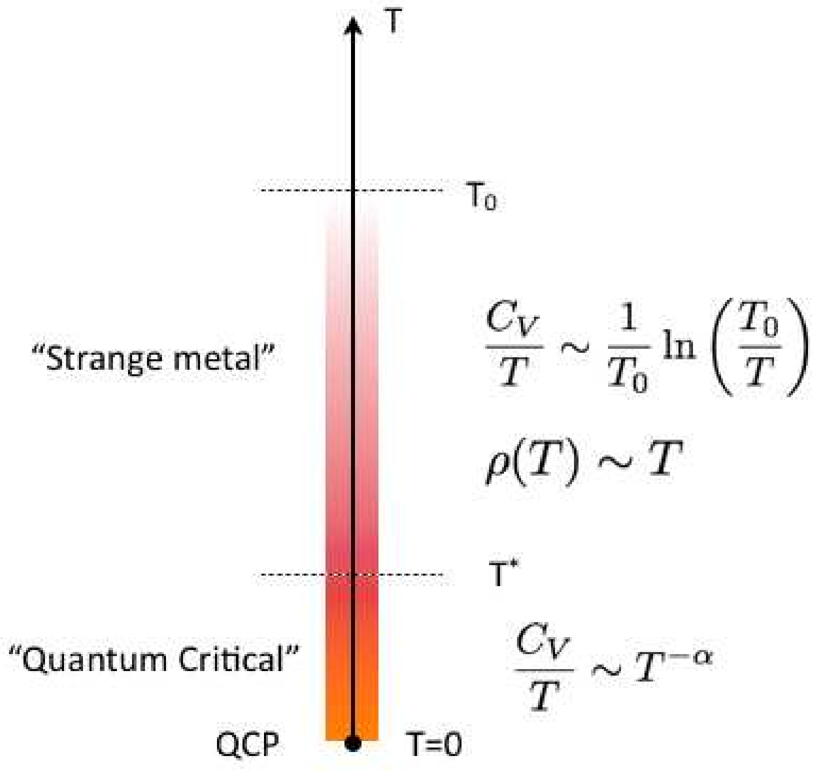

Figure 1 The cross-over between "strange metal" behavior that occurs at high temperatures, to "quantum critical behavior" in the vicinity of the quantum critical point.

Suchitra Sebastian asked *what do unconventional power laws really mean and with what theory should one compare them?* It is well know that in the approach to a classical critical point, that larger temperature deviations $T - T_c$ from the critical point are governed by Ginzburg Landau theory and true critical behavior only develops once the Ginzburg criterion is violated. Could a cross-over between two different types of behavior be at work in quantum criticality too?

Kazuo Ueda remarked that a key observation in f-electron systems, is that the mass of electron quasiparticles becomes very heavy near a quantum critical point, and that as the characteristic Fermi temperature collapses, strange metal behavior is seen to develop. In addition to spin fluctuations, he said, there are a variety of other slow quantum fluctuations that may be important in driving up quasiparticle mass, including local orbital fluctuations and anharmonic lattice vibrations. Two groups in the new consortium will explore such new mechanisms, he said. One of the guiding principles that researchers may follow here, is to look for systems where the magnetic ion lies at a point of high symmetry.

Brian Maple expanded further on the need for a more general exploration of quantum criticality. He argued that the view that the identification of strange metal physics with a bulk quantum critical point may be too restrictive. Indeed, key anomalous characteristics of strange metal behavior, such as anomalous temperature dependence in

– resistivity, $\rho(T) \sim \rho_0 \pm AT^n$ ($1 \leq n \leq 1.5$ with n usually close to 1).
– specific heat $C(T)/T \sim -lnT,\ T^{-n}$, ($n \sim 0.2 - 0.4$) and
– magnetic susceptibility $\chi(T) \sim -lnT,\ T^{-n}$ ($n \sim 0.2 - 0.4$), along with
– the observation of ω/T scaling in neutron scattering with $\chi''(q,\omega) \sim f(\omega/T)$.

are found in widely different circumstance. Not only only are they found near bulk antiferromagnetic quantum critical points, as in $YbRh_2Si_2$, $CeRhIn_5$ under pressure, and $CeRh_{1-x}Co_x\ In_5$, they are also observed at spin-glass quantum critical points in $U_{1-x}\ Y_xPd_3Al_2$, $UCu_{5-x}Pd_x$, near ferromagnetic quantum critical points (in $URu_{2-x}Re_xSi_2$ and $CePd_{1-x}Rh_x$). Moreover, he noted, they are also observed far from any readily identifiable bulk quantum critical point, as in ($Y_{1-x}U_xPd_3$, $Sc_{1-x}U_xPd_3$ and $U_{1-x}Th_xPd_3Al_2$) prompting a speculation that there might be a single ion, local quantum-critical character. These observations led Brian Maple to ask:

Is there a more general scenario that encompasses these situations and presently proposed mechanisms (e.g., Kondo disorder, quadrupolar Kondo, Griffiths phase, 2nd

order AFM, SG, or FM transition suppressed to 0 K, etc.)?

As an example, Maple noted $Y_{1-x}U_xPd_3$, the first f-electron material in which non-Fermi liquid behavior was observed), $Sc_{1-x}U_xPd_3$, and URu_2Si_2, where there is evidence that the magnetic uranium ion is tetra-valent ($U^{4+},\ 5f^2$), with two localized f-electrons. In this situation, there is the possibility of a Γ_3 or Γ_5 non-Kramers doublet ground state, setting the stage for a quadrupolar Kondo effect, originally proposed for $Y_{1-x}U_xPd_3$. Perhaps, he proposed, this scenario, modified to account for interactions between U ions, can, after all, account for NFL behavior in these systems.

4 The mechanism of quantum criticality. The discussion turned to the mechanism of quantum criticality as observed in f-electron materials. One of the subjects of particular interest, concerned the evolution of the Fermi surface through a quantum critical point. **Meigan Aronson** discussed how measurements have documented the extent to which the critical fluctuations associated with the $T = 0$ K transition affect a wide range of measured quantities, magnetic, thermal, and transport. Yet at the same time, she noted, various experiments such as Hall conductivity, neutron and de Haas van Alphen suggest the f-electron may be localizing at a magnetic heavy electron quantum critical point, rasing her to pose the key question:

> *how do the anomalous fluctuations at a quantum critical point impact the underlying electronic structure and give rise to the apparent f-electron delocalization transition that has been found to exist at or near the quantum critical point in some systems?*

The possibility of a class of quantum phase transitions where the Fermi surface jumps in area or volume was taken up in detail by **Todadri Senthil**. Senthil cited both heavy fermion Quantum Critical Points and Mott Metal-insulator transitions (which may occur in under and over-doped cuprate superconductors) as possible examples. He argued that since these transitions appear to be second-order, non-Fermi liquid physics follows very naturally at such a QCP, but that the intuition built up from "bosonic quantum criticality" (electrons coupled to a fluctuating order parameter) would not be relevant.

Senthil described how a model of "quantum critical Fermi surfaces" is appropriate to describe the paradoxical combination of a first-order jump in the Fermi surface at a second-order phase transition[12]. According to this picture, the large and small Fermi surface co-exist at a QCP, while the jump Z in momentum-space occupancy is expected to vanish[5,13] at the QCP to be replaced by a Fermi surface with a power-law singularity in the electron Green functions, similar to what is found in a one-dimensional Luttinger Liquid (Fig. 2).

One of the interesting questions raised by Senthil, is whether

> *the jump in the Fermi surface volume and the development of magnetic order are necessarily linked to one-another, or whether are they two different phenomenon?*

Piers Coleman described how [14] insights from quantum magnetism and recent experiments tend to support this point of view. Imagine he said, connecting a frustrated antiferromagnet to a conduction sea via a tunable Kondo interaction. Various groups[15,16,17,18] have considered a two-dimensional phase diagram with $x-$ordinate describing the tuning $K = T_K/J_H$ of the ratio between the Kondo temperature and the nearest-neighbor RKKY interaction, and $y-$ordinate describing the intensity $Q-$ of antiferromagnetic quantum zero-point fluctuations (which can be tuned for example, by increasing the amount of frustration). While there there is a common antiferromagnetic phase at small K and Q, the the paramagnetic "spin liquid" at large Q has a small Fermi surface, while the the paramagnetic heavy Fermi liquid at large K has a large Fermi, suggesting that the two are separated by zero-temperature phase transition (Fig. 3). The existence of this transition appears to have been observed in field-tuning experiments on Ir and Ge doped $YbRh_2Si_2$ ($YbRh_{2-x}Ir_xSi_2$[19] and $YbRh_2Si_{2-x}Ge_x$[18]. In these materials, there is a field-tuned temperature scale T^*(B) where various anomalies are seen in the Hall constant, susceptibility and Grüneisen parameter are seen to sharpen up at the critical field B_c where $T^*(B_c) \rightarrow\ 0$. This point has been interpreted as the point of field-tuned transition between a small and a large Fermi surface. **Philipp Gegenwart** pointed out that in $YbRh_2Si_2$, the field-tuned T^* scale and the Néel temperature line and T_N(B) converge at a single quantum critical point but that they separate in Ir or Co or Ge doped systems[19,18]. Similar features, though less intensively studied, are seen at higher magnetic fields in YbGeSi[20]. In both types of material, a strange metal phase appears to lie lie between the ordered antiferromagnet and the heavy electron state. **Philipp Gegenwart** raised various questions about these ideas. He asked,

– *what is the coincidence of scales in undoped $YbRh_2Si_2$ that brings the Néel and "T^* line" together?,*
– *What is the nature of the "spin liquid phase" that is predicted to develop in Ir doped $YbRh_2Si_2$?*
– *In YRS, why does pressure fail to influence the "T^* line"? [21]*

Senthil had several points to make about this kind of phase diagram, which he epitomized by the phrase *"Quantum is different"* . These important differences can occur in many different guises. For example, from from work on frustrated two-dimensional antiferromagnets, there are indications that the continuous QPT from antiferromagnet to spin liquid, or valence bond solid may involve a new kind of "deconfined criticality" with emergent fractional degrees of freedom[22]. He also mentioned the possibility of topological order[23,24] and *"self organized criticality"*

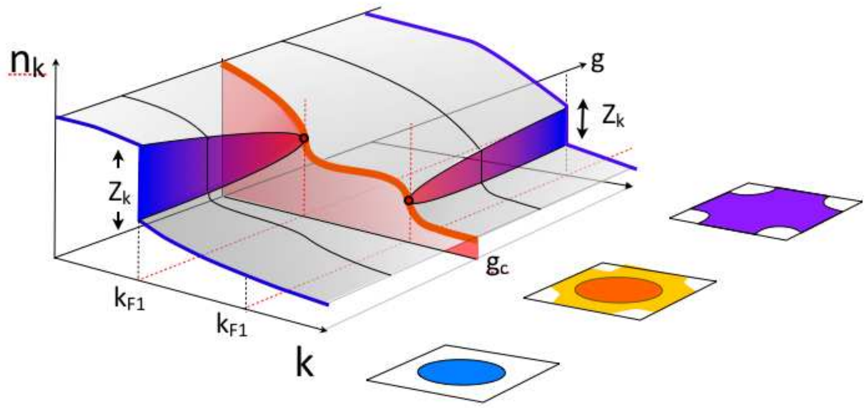

Figure 2 (Color online) Schematic illustrating Senthil's "quantum critical Fermi surface" scenario[12], whereby the jump in the occupancy Z_k at the small Fermi surface vanishes at the quantum critical point[5], leaving behind a power-law singularity. As the tuning parameter passes beyond its critical value g_c, a new "large" Fermi surface develops at $k = k_{F2}$, describing the heavy Fermi liquid.

where strange metal phases containing algebraic order in space, or time, might develop without fine-tuning to a QCP, such as a critical spin liquid phase[25, 26].

Philipp Gegenwart, **Hilbert von Lohneysen** and **Brian Maple** turned the subject towards the practical, experimental classification and characterization of different types of quantum critical point.

Gegenwart described how antiferromagnetic quantum critical points in metals appear to divide into two classes- "conventional", spin-density wave transitions and "unconventional" QCP's where the physics appears more localized. One of the most useful methods to delineate between different classes of quantum critical point, he said, is the Grüneisen parameter, which measures the ratio

$$\Gamma = \frac{V^{-1}dV/dT}{C_P}.$$

Under the assumption that the free energy contains a single energy scale, so that $F(T) \sim T\phi(T/T^*(P))$, $\Gamma \propto \frac{d\ln(T^*)}{dP}$. It has been shown by scaling arguments[27] that Γ diverges in the approach to any QCP. For QCPs of the SDW type, the critical Grüneisen ratio diverges as $1/T$. By contrast, a divergence with fractional exponent ($\Gamma \sim T^{-0.7}$) has been found in Ge-doped $YbRh_2Si_2$ [28], which appears to be compatible with the predictions of a locally quantum critical senario. If Γ has no divergence, one can exclude a generic QCP as the origin of non-Fermi liquid behavior. Gegenwart noted that experimentally, both types of QCP (SDW and local) are observed in different heavy fermion metals. Unconventional transitions pose the greatest challenge, with many open questions:

– Which types of unconventional exist?
– What are the conditions they arise from?
– Which observed features are generic and which are material specific?

Gegenwart turned to discuss the field-tuned quantum criticality of of $YbRh_2Si_2$, one where the magnetic Grüneisen parameter $\Gamma_M = -\frac{dM/dT}{C_H}$ diverges as $\Gamma_M = G_r/(H - H_c)$ with $G_r = -0.3$[29]. Gegenwart asked whether this result might be consistent with the critical Fermi surface model of Senthil?

Hilbert von Lohneysen emphasized that the difficulty with any kind of global phase diagram, is that we do not understand the roles of different tuning parameters at a QPT. He pointed out that there are only few systems where different parameters have been employed to tune a QCP. In the case of $CeCu_{6-x}Au_x$, three different tuning parameters have been employed: chemical doping with gold, concentration x, hydrostatic pressure P, and magnetic field B. While P and x tuning lead to the same T dependencies in resistivity ρ and specific heat C, both suggestive of a local QCP, field tuning shows dependencies in ρ and C that are more indicative of a spin-density wave (SDW)-type QCP [2]. The different scaling behavior for x and B tuning has been corroborated by inelastic neutron scattering experiments, directly measuring the critical fluctuations [1, 3]. These experiments put definite constraints on the construction of a multi-parameter zero-temperature (global) phase diagram. More experiments along theses lines are certainly needed.

Brian Maple discussed how little we understand about ferromagnetic quantum critical points in $f-$electron systems. Unfortunately, he said, most ferromagnetic systems exhibit a first order QPT under pressure, but a second order QCP under chemical doping. He introduced $URu_{2-x}Re_xSi_2$ as a fascinating new development in this respect. Undoped URu_2Si_2 exhibits "hidden order", but on doping with rhenium, he said, the transition temperature of the hidden order is continuously suppressed to zero at $x_c \approx 0.15$. The QPT where the hidden order disappears coincides with a sudden appearance of ferromagnetism, with a Curie temperature T_c that grows linearly with $x-x_c$. This ferromagnetic phase exhibits non-Fermi liquid properties: a logarithmic temperature dependence of the specific heat and an anomalous temperature dependence of the resis-

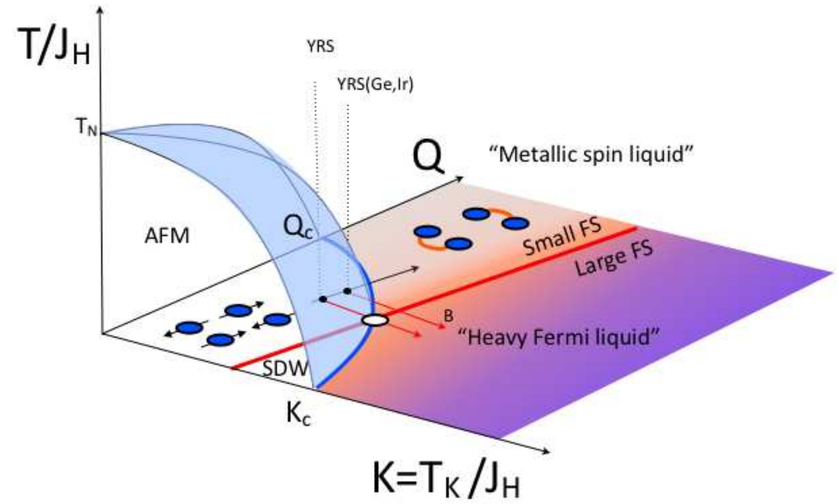

Figure 3 (Color online) Schematic "global phase diagram" for heavy fermion materials, adapted from reference [18], showing "Kondo axis" tuning the ratio $K = T_K/J_H$ of the Kondo temperature to Heisenberg interaction versus the "Quantum axis" tuning the strength of magnetic zero point fluctuations Q. Red line denotes zero temperature quantum phase transition between small and large Fermi surface states. Labeled are the hypothetical locations of stoichiometric $YbRh_2Si_2$(YRS) and the same compound, doped with Ge or Ir (YRS (Ge,Ir)), showing presumed effect of magnetic field and doping.

tivity. The logarithmic specific heat exhibits a remarkable parabolic dependence on doping,

$$\gamma(x,T) \sim -(x-x_c)(x_2-x) \times \frac{1}{T_0} \ln T,$$

where $x = x_2 = 0.6$, even though ferromagnetism continues to higher doping. Maple reported that a careful study of evolution of the temperature exponent of the susceptibility $\chi \sim t^{-\gamma}$, the field and temperature dependence of the magnetization $M \sim H^{1/\delta},\ t^{\beta}$ (linked by the relation $\delta - 1 = \gamma/\beta$) shows that as a function of doping, γ and δ grow linearly with $x - x_c$ while $\beta \approx 1$ is constant[30]. This fascinating physics awaits a theory.

Tomo Uemura and **Suchitra Sebastian** both returned the discussion of the nature of the soft quantum modes near a QPT. Both raised the issue of the relationship of spin and charge degrees of freedom in materials close to a quantum critical point, and indeed, whether spin is the most important slow degree of freedom.

Tomo Uemura raised two interesting points in connection with the soft-modes at a QCP. First, he asked, what is the role of importance of first-order quantum phase transitions. Even though many quantum phase transitions are first order, they may still exhibit a variety of important soft modes that while strictly speaking, remain gapped at the transition, still influence the physics over a wide finite temperature range. Such soft modes are the analog of "roton modes" in He^4, but they might occur in the spin, the charge and even the phase channel. In general, even if the QCP is first order, the energy of such soft modes can be used as an indicator of the closeness to a competing state[31, 32].

Uemura speculatively introduced the idea of "resonant spin-charge coupling" - while most of us think of the charge and spin modes of strongly correlated systems as separate degrees of freedom,

> *is it possible, he mused, that that slow spin and charge modes become resonantly coupled?*

Uemura he showed how the superconducting transition temperature T_c in a wide range of unconventional superconductors scales with the Fermi temperature T_F (obtained from the linear specific heat) and the spin-fluctuation scale (obtained from the magnetic susceptibility), namely

$$T_c \propto T_F,\ T_{SF}.$$

Such scaling relationships seem to hold over many decades of variation in T_c. Conventionally, these kinds of scaling relationships are interpreted in terms of an anisotropic pairing within a Fermi liquid, in which the a single renormalized Fermi temperature of the pairing electrons also governs the characteristic spin fluctuation scale. Uemura argues that an alternative way to interpret tracking between T_F and T_{SF} illustrates a resonance between spin and charge fluctuation modes.

Suchitra Sebastian viewed the problem from another perspective. She asked:

> *should we think of a separation of charge and spin Quantum critical points?*

Such a separation has, she pointed out, been used to understand the effect of pressure in $CeCu_2Si_{2-x}Ge_x$, where the superconducting transition temperature is seen to exhibit two separate maxima as a function of pressure - a lower pressure maximum in the vicinity of a magnetic plus a higher pressure maximum in the vicinity of a valence instability of the material[33, 34]. But could this, she asked, be part of a much more general phenomenon? Could one, for example, understand the two transitions seen in Ge and Ir doped $YbRh_2Si_2$ as a spin and a charge critical point?[19, 18]. Sebastian also showed de Haas van Alphen measurements on $CeIn_3$, in where a field-induced quantum phase transition is observed within the antiferromagnetic phase

at which the effective mass of the quasiparticles diverges while the orbit areas collapse to zero[35]. A similar divergence in quasiparticle effective masses has also been recently seen in the de Haas van Alphen measurements on under-doped $YBa_2Cu_3O_{6+x}$[36]. Could these, she asked, be further examples of this general phenomenon - QPT involving charge?

5 New Phases, superconductivity and the d-f connection Throughout the discussion, physicists constantly returned to the theme of superconductivity, its connection with quantum criticality and the usefulness of f-electron research as a platform for understanding ordering phenomenon at higher energy scales, d-electron transition metals. The “d-f” connection was discussed by several of the panelists.

Tomo Uemura discussed how the broad trends in T_c with superfluid density which cut across anomalous f- and d-electron superconductors support the idea that we should endeavor to understand these phenomenon within a unified framework. As part of this framework, he argued, there are two extreme limits to consider - one extreme - that of BCS pairing, while on the other - that of Bose-Einstein condensation of pre-formed pairs. Uemura argues that the scaling of T_c with superfluid density is an indication that anomalous superconductivity lies at the BCS-BEC cross-over between these two extreme regimes.

Kazuo Ueda Discussed how a major part of the new Japanese f-electron consortium, is the discovery of new materials, with new types of broken symmetry groundstate will broaden our understanding of strong correlation, helping the d-f connection. This thrust has two components, he said -

– By extending the search for novel f-electron behavior along the rare earth series - at one end of the series, from Cerium to Paladium compounds, and at the other end of the series, from Ytterbium to Thullium (Tm) materials.
– By pushing up to higher energy scales, intermediate between the 4f- and 3d materials through the exploration of 5f correlated electron materials. By going from the Cerium 115 materials to related transuranic materials, $PuCoGa_5$ and $NpAl_2Pd_5$, it has proved possible to substantially raise T_c. Are there other examples of this trend?

Meigan Aronson discussed the difficulties in making the d-f connection, noting that while an extensive body of measurements on itinerant ferromagnets such as $ZrZn_2$ and MnSi, led the way in establishing magnetic quantum criticality in a d-electron context, [37,38], unlike their f-counterparts, these systems have ‘large’ Fermi surfaces in both the ordered and paramagnetic regimes, where the d-electrons are included in the Fermi surface [39]. While systems such as V_2O_3 [40] and $Ni(S_{1-x}Se_x)_2$ [41] are considered exemplars of Mott-Hubbard physics, with a first order transition from strongly correlated metal to localized moment insulator [42], she points out we still have not discovered a transition metal compound that is the analog of magnetic field tuned $YbRh_2Si_2$ [43] or pressurized $CeRhIn_5$[7], where quantum critical points are the source of both strong quantum critical fluctuations and transitions where an f-electron is delocalized, driving the Fermi surface from small to large[44].

6 Conclusion The panel discussion prompted lively debate throughout the QCNP09 meeting, and appears to provide a useful model for future scientific conferences of this time. The study of quantum criticality and its intimate relationship with material physics and the emergent quantum mechanics of the periodic table, make it an area of burgeoning discovery. As a reporter on this event, I’d like end with a quote from Meigan Aronson at this event, on the prospects of a future d-f connection surrounding quantum criticality and high temperature superconductivity (see Fig. 4):

> *“So far, our interest has focused on ferromagnetic systems, which are ultimately discontinuous, and not quantum critical, when the Curie temperature becomes sufficiently low [45,46]. Whether a similar behavior will be found in (d-electron) quantum critical antiferromagnets remains an interesting and controversial question, little explored due to a lack of suitable host systems. Still, these are the compounds in which unconventional superconductivity is presumed to be most likely, particularly if it is possible to realize the strongly correlated state found on the metallic side of a Mott-Hubbard transition. For these reasons, it seems more pressing than ever to refresh our interest in intermetallic compounds where the magnetic entity derives from transition metal moments.”*

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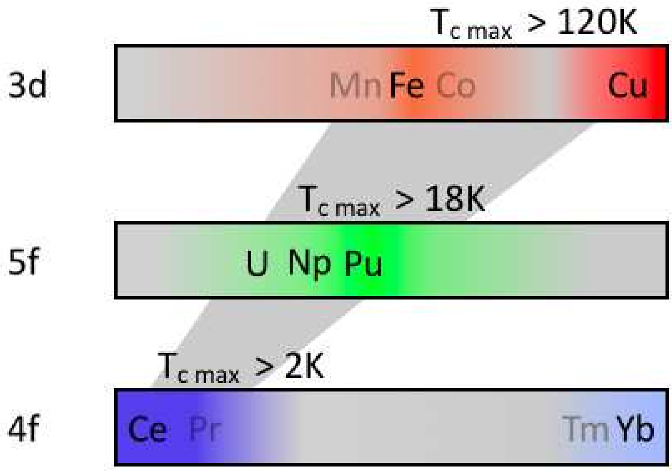

Figure 4 (Color online). The "d-f" connection. As one moves from 4f to 5f through 3d compounds, the characteristic scale of the d/f electrons rises, and with it the maximum observed superconducting temperature. Shaded elements in each series denote the d/f metals whose compounds are give the currently highest superconducting T_c.

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